Lecture 1: Commodity Taxation and Production Efficiency

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Constraints vs Instruments

- **2nd Welfare Theorem**: Any Pareto Efficient outcome can be reached by;
  1. Suitable redistribution of initial endowments [individualized lump-sum taxes based on indiv. characteristics and not behavior],
  2. Then letting markets work freely
- But constraints on communication, data, and administration of an economy (not to mention political constraints) limit possibilities of **First-Best Taxation**.
- Two approaches:
  1. **Incorporate constraints** into economic theory of government redistribution. [Mirrlees 1971]
  2. View constraints as **ad-hoc limits on sets of policy tools** available to government. [Ramsey, Diamond-Mirrlees I, II, TODAY]

Overview of Today's lecture

- Optimal commodity taxation and Production Efficiency: Three approaches
    1. Geometric Approach: Edgeworth Box
    2. Primal Approach: choose allocations
    3. Dual Approach: chooses taxes/prices directly
        - Uniform Taxation
        - Inverse Elasticity Rule and Caveats
        - Elasticity of Income vs Price Elasticy
        - Muti-Agent Dual Problem

Productive Efficiency (Diamond-Mirrlees)

- P. Diamond and J. Mirrlees, "Optimal Taxation and Public Production I: Production Efficiency," and "Optimal Taxation and Public Production", American Economic Review 61 (1971), 8-27 and 261-278.
- Many consumers (H), many goods (N) and inputs.
- Fixed vector of government expenditures.
- Important assumptions:
  - constant returns to scale (or fully taxed profits).
  - full set of differentiated taxes on inputs and outputs.

First-Best Problem

- Consumer prices q.
- Consumers

$$\max_{x^h} u^h(x^h) \quad \sum_{i \le N} q_i x_i^h \le I$$

  - indirect utility function $V^h(q, I)$ and uncompendated (Marshallian) demand functions $x^h(q, I)$
  - write $V^h(q) = V^h(q, 0)$ and $x^h(q) = x^h(q, 0)$ for short.
  - we could have $u^h(x^h, g)$, but in what follows g is fixed, so we suppress the dependence.
- Government consumption g.
- Production function (intermediate goods suppressed)

$$F(y) \le 0 \quad y = x(q) + g$$

  where $x(q) = \sum_{h \le H} x^h(q)$.
- First-best: $MRS_{ij}^h = MRS_{ij}^{h'}$, $\quad MRS_{ij}^h = MRT_{ij}$, and $F = 0$ (efficient production).
- The first-best can be achieved with a lump sum tax but not without a lump sum tax.

Second-Best Problem

- The second best problem is to choose an allocation and production plan subject to the requirement that the allocation must be supported by an equilibrium price vector q:

$$\max_{q} \lambda^h V^h(q) \quad F(x(q)+g) \leq 0$$

- Productive efficiency: at the optimum, the allocation is on the Production Possibility Frontier (PPF)

$$F(x(q)+g) = 0$$

- Result can be stated algebraically using MRS and MRT. Consider two firms a and b (potentially in different industries) and two inputs K and L, then

$$MRT^a_{KL} = MRT^b_{KL}$$

even if the allocation is not Pareto efficient

$$MRT_{KL} \neq MRS_{KL}$$

- Productive efficiency can be achieved by letting firms face prices producer prices p so that firms solve

$$\max_{y} py \quad F(y) \leq 0$$

and then taxes are just given by $\tau = q - p$.

Implications of Productive Efficiency

- **Public sector production should be efficient.** If the public sector is producing some goods, it should face the same prices p as the private sector, and it should choose production with the unique goal of maximizing profits.

- **Intermediate goods** (that are neither direct inputs not outputs to individual consumption) **should not be taxed**. Taxes on transactions between firms would disrupt productive efficiency. Ex: computer sales to firms should be untaxed, but computer sales to consumers should be taxed.

- **Trade:** In a small open economy, the production set is extended because it is possible to trade at linear prices with other countries. The productive efficiency result implies that the small open economy should be on the extended PPF. **No tariff should be imposed on goods and inputs imported or exported by the production sector.** Ex: computer sales by domestic firms to foreigners should be untaxed, purchases of computers from other countries should be untaxed, and their should be no special tariff on foreign computers compared to domestic computers.

Single Agent Ramsey Problem: Geometric Approach

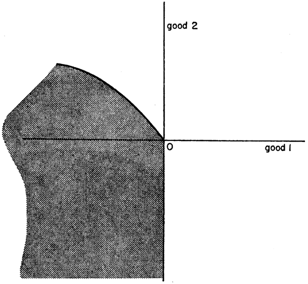

FIGURE 1

- Single agent
- in general assume private production is CRS. Here, we assume all production possibilities controlled by government.
- think of good 1 as labor and good 2 as a consumption good.

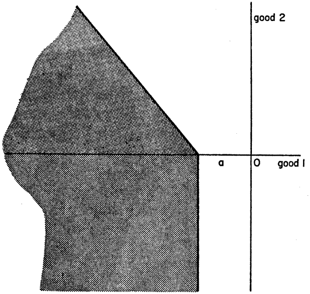

FIGURE 2

- with DRS government production, or CRS + fixed expenditures a (such as defense), a poll tax (lump sum tax with single agent) can achieve Pareto optimum (maximizes agent's utility in single-agent economy)

- Poll tax unreasonable in multi-agent economy. We restrict instrument to the use of linear commodity taxes: the planner can only deal with consumers through the market place (equilibrium) and set the price of the consumer good relative to the wage.

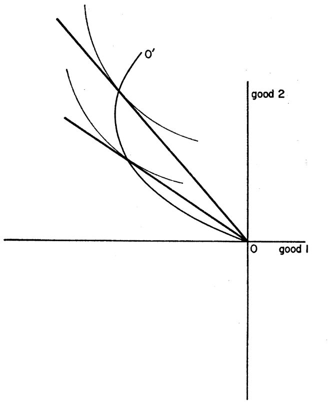

FIGURE 3

- Consumption bundles which the consumer is willing to achieve by trade from the origin is the offer curve OO'
- Consumer indifference curves at different budget lines

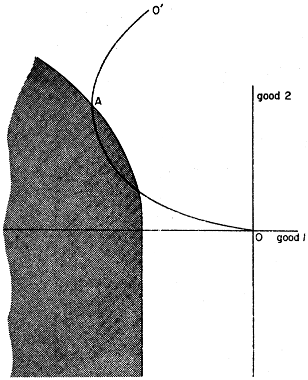

FIGURE 4

- Bold line: range of consumption bundles which are both feasible and potential consumer equilibria
- If social and individual welfare coincide, we wish to move far along OO' while staying in production possibility set.

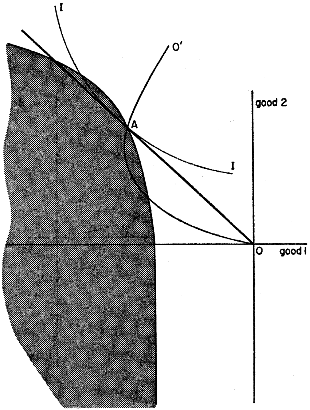

FIGURE 5

- Optimum: Point A
- Budget line: OA, defines relative price and commodity taxes.
- Indifference curve: II'. All the points above II' and in the shaded production set are Pareto superior to A and technologically feasible, but not attainable by market transactions without lump sum transfers.

Competitive Equilibrum

Definition: A competitive Equilibrium (CE) with taxes is p, q, x is such that:

1. x solves the consumer's maximization problem

$$\max_x u(x) \quad \sum_{i \leq N} q_i x_i \leq 0$$

e.g $u(c_1, c_2, \ldots, c_{N-1}, l)$ and $\sum_{i \leq N-1} p_i(1+\tau_i)c_i = (1-\tau^l)wl$

2. y solves the firms' profit maximization problem (CRS e.g. $\sum_{i \leq N-1} p_i y_i - l \leq 0$)

$$\max_y py \quad F(y) \leq 0$$

3. x, g, t, p satisfy the government budget constraint

$$\sum_{i \leq N} p_i g_i \leq \sum_{i \leq N} t_i x_i$$

4. Markets clear:

$$x_i + g_i = y_i, \forall i \leq N$$

- Result of **Market Clearing + Walras' Law**: CE $\iff F(x+g) = 0$ and agent optimization (1). Note that the second condition involves x and q only.

Primal and Dual

- First-Best

$$\max_x u(x) \quad F(x+g)=0$$

- Second-Best

$$\max_{x,q} u(x) \quad F(x+g)=0$$

$$x \in \arg\max_x u(x) \quad \sum_{i\leq N} q_i x_i \leq 0$$

1. "Primal approach": choose allocations x. Solve q as a function of x
   - pros: "fool-proof" method for finding optimal allocations in terms of exogenous variables, conceptually easy to extend to complicated settings (dynamics, uncertainty)
   - cons: intuition is often less clear, results obtained in terms of second derivatives and cross-partials of the utility functions which may be hard to estimate empirically
2. "Dual approach" choose taxes τ directly. Solve x as a function of q.
   - pros: get all expressions in terms of elasticities, which have clear economic interpretations; intution is often clear; easy to connect to empirical work
   - cons: elasticities are usually endogenous to a particular tax system, so ultimately get expressions for taxes in terms of endogenous objects; hard to extend the analysis beyond simple settings

Dual Approach

- Second-Best: [$x(q)$ Marshallian/uncompensated demand]

$$\max_q V(q) \quad F(x(q)+g)=0$$

- First order condition

$$\frac{\partial V}{\partial q_j} - \lambda \sum_i \frac{\partial F}{\partial y_i}\frac{\partial x_i}{\partial q_j} = 0$$

- By Roy's identity, profit maximization, and Slutsky equation:

$$\frac{\partial V}{\partial q_j} = -\alpha x_j, \quad p_i = \frac{\partial F}{\partial y_i}, \quad \frac{\partial x_i}{\partial q_j} = \frac{\partial h_i}{\partial q_j} - x_j \frac{\partial x_i}{\partial I}$$

, where $\alpha = \frac{\partial V}{\partial I}$ and h is the Hicksian/compensated demand function $h(q, V(q)) = x(q)$

- Replace all 3:

$$-\frac{\alpha}{\lambda} x_j - \sum_i p_i \frac{\partial h_i}{\partial q_j} + x_j \sum_i p_i \frac{\partial x_i}{\partial I} = 0$$

Dual Approach

- We know $\sum_i q_i \frac{\partial h_j}{\partial q_i} = 0$ and by symmetry of the Slutsky matrix $\frac{\partial h_j}{\partial q_i} = \frac{\partial h_i}{\partial q_j}$ so

$$-\sum_i p_i \frac{\partial h_i}{\partial q_j} = \sum_i t_i \frac{\partial h_i}{\partial q_j}$$

- We also know that $\sum_i q_i \frac{\partial x_i}{\partial I} = 1$ so

$$\sum_i p_i \frac{\partial x_i}{\partial I} = 1 - \sum_i t_i \frac{\partial x_i}{\partial I}$$

- Thus

$$\sum_i t_i \frac{\partial h_i}{\partial q_j} = -x_j \theta$$

where [independent of i, j]

$$\theta = -\frac{\alpha}{\lambda} + 1 - \sum_i t_i \frac{\partial x_i}{\partial I}$$

- Multiplying by t_j and summing

$$\theta \sum_t x_j t_t = -\sum_{i,j} t_i \frac{\partial h_i}{\partial q_j} t_j \geq 0$$

by negative semi-definiteness of Slutsky matrix. So θ has the same sign as gov revenue.

Interpretation: "Uniform Taxation"

- By symmetry

$$\frac{1}{x_j}\sum_i t_i \frac{\partial h_j}{\partial q_i} = -\theta$$

- the percentage change in the demand for goods j from tax change are the same
  - Evaluated at constant production prices
  - consumer were compensated so as to stay on the same indifference curve
  - derivatives of the compensated demand curves were constant at the same level as at the optimum
- Uniform θ interpreted (falsely) as an estimate of how much good x_j fell due to taxation: flavor of uniform taxation
- Actual changes in uncompensated demand differ from proportionality with a larger than average percentage fall in demand for goods with a large income derivative

$$\frac{1}{x_j}\sum_i t_i \frac{\partial x_j}{\partial q_i} = -\theta - x_j^{-1}\frac{\partial x_j}{\partial I}\sum_i t_i x_i$$

: flavor of high tax on necessities

Interpretation: "Inverse Elasticity Rule"

- With elasticities

$$\sum_i \frac{t_i}{p_i + t_i} \varepsilon_{ij}^c = \theta$$

- Important implication of tax formula: elasticities of substitution **in production do not matter.** Incidence in the production sector and general equilibrium responses can be ignored in the formulas. As a result, oftentimes in optimal taxation papers, one assumes linear technologies (not restrictive, replace the p with the p that arises in equilibrium).

- Special case, when $\varepsilon_{ij}^c = 0$ for $i \neq j$ so that the Slutsky matrix is diagonal, we obtain a classic inverse elasticity rule

$$\frac{t_i}{p_i + t_i} = \frac{\theta}{\varepsilon_{ii}^c}$$

- The Inverse Elasticity Rule however requires strong assumptions like no price cross-complementary between goods.

Literature's take away

- "To summarize, with additive separability, the general result is that tax rates depend on income elasticities, with necessities taxed more than luxuries. Moreover, the familiar intuition from partial equilibrium that goods with low price elasticities should be taxed heavily does not necessarily apply in a general equilibrium setting." (Chari and Kehoe 1999, p. 1681-82)
- "When utility function is additively separable, the optimal tax rate depends inversely on the income elasticity of demand. This clearly has important implications for the confliict between equity and efficiency" (Atkinson and Stiglitz, 1972, p. 109)

Price Elasticities and Income Elasticities

- **(Pigou, Deaton)**: If preferences are additively separable, income elasticities are proportional to prices elasticities
- So assumption, the good is price inelastic if it is a necessity
- Empirically not true
- **Exercice:** Consider preferences in Hanoch (1975), Lashkari-Mestieri (2016) $U(\{c_i\}, l) = U(C) - l$ where C is the implicit function of $\{c_i\}_i$

$$\sum_i \Omega_i^{1/\sigma_i} C^{(\epsilon_i - \sigma_i)/\sigma_i} c_i^{(\sigma_i - 1)/\sigma_i} = 1$$

  . This preference gives σ_i price elasticity and ϵ_i income elasticity and cross-price elasticity proportional to σ_i/σ_j. Show that $t_i \geq t_j$ if $\sigma_i \leq \sigma_j$ (tax price inelastic goods at a higher rate) or $\epsilon_i \geq \epsilon_j$ (tax luxuries at a higher rate).
- Once the link between income and price elasticities is broken, the prediction run counter to conventional wisdom.

Primal Approach

- Primal solves q as a function of x
- Consumer optimization

$$x \in \arg\max_x u(x) \quad \sum_{i \leq N} q_i x_i \leq 0$$

- Necessary and sufficient conditions: $\exists \nu > 0$ s.t. (assuming local non-satiation)

$$q_i = \nu u_i(x) \quad \sum_{i \leq N} q_i x_i = 0$$

thus we have the **implementability condition**

$$\sum_{i \leq N} u_i(x) x_i = 0$$

- Reverse is also true: if $\sum_{i \leq N} u_i(x) x_i = 0$ then $\exists q$ such that $x \in \arg\max_x u(x), q \cdot x \leq 0$
- Second-best

$$\max_x u(x), \quad \text{s.t. } F(x+g) = 0, \quad \text{and } \sum_{i \leq N} u_i(x) x_i = 0$$

Primal Approach

- Lagrangian:

$$L = u(x) + \mu \sum u_i(x) x_i - \gamma F(x+g)$$

- FOC

$$(1+\mu) u_i(x) + \mu \sum_j u_{ij}(x) x_j = \gamma F_i(x+g)$$

- Implication

$$\frac{F_i(x+g)}{F_k(x+g)} = \frac{u_i(x)}{u_k(x)} \frac{1+\mu+\mu\sum_j \frac{u_{ij}(x)}{u_i(x)} x_j}{1+\mu+\mu\sum_j \frac{u_{kj}(x)}{u_k(x)} x_j}$$

- Since

$$\frac{F_i(x+g)}{F_k(x+g)} = \frac{p_i}{p_k}, \quad \frac{u_i(x)}{u_k(x)} = \frac{q_i}{q_k}$$

- Tax rate (where $q_i = (1+\tau_i) p_i$)

$$\frac{1+\tau_k}{1+\tau_i} = \frac{1+\mu+\mu\sum_j \frac{u_{ij}(x)}{u_i(x)} x_j}{1+\mu+\mu\sum_j \frac{u_{kj}(x)}{u_k(x)} x_j}$$

Uniform Taxation

- **Exercice:** A uniform commodity taxation result: if $u(G(x_1, x_2, \ldots, x_{N-1}), l)$ and G is homogeneous of degree 1 then $\tau_1 = \tau_2 = \cdots = \tau_{N-1}$.

- The proof uses $G = \sum G_i x_i$ to derive

$$\frac{u_{ij}}{u_i} x_j = \frac{u_{GG} G_j x_j}{u_G} + \frac{G_{ij} x_j}{G_i}$$

$$\frac{u_{il}}{u_i} x_l = \frac{u_{Gl} x_l}{u_G}$$

and finally

$$\sum_j \frac{u_{ij}}{u_i} x_j = \frac{u_{GG} G}{u_G} + \frac{G_i + \sum_j G_{ij} x_j}{G_i} - 1 = \frac{u_{GG} G}{u_G} - 1$$

- Homothetic preferences benchmark, more useful than "inverse elasticity rule": Uniform taxation irrespective of price elasticities.

Multi-Agent Dual

- P. Diamond, "A Many-Person Ramsey Tax Rule," Journal of Public Economics 4 (1975), 335-342.
- Second Best (dual)

$$\max_{q,I} \sum_h \lambda^h \pi^h V^h(q, I) \quad \text{s.t} \quad F(\sum_h \pi^h x^h(q, I) + g) = 0$$

- Note about wealth I
  - we can impose $I = 0$
  - typically we do not want to: captures a lump sum transfer/tax
  - if we allow I freely chosen by planner then productive efficiency is obvious
- more generally
  - Pareto problem not convex
  - cannot maximize weighted utility
  - but pareto weights for local optimality condition

Multi-Agent Dual

- Define Lagrangian

$$L = \sum_h \lambda^h \pi^h V^h(q, I) - \lambda F(\sum_h \pi^h x^h(q, I) + g)$$

- FOC (using same identities as in single-agent case)

$$\sum_h \lambda^h \pi^h \frac{\partial V^h}{\partial I} - \lambda \sum_{h,i} \pi^h F_i \frac{\partial x_i^h}{\partial I} = 0$$

- notation:
  - population average: $\mathbb{E}_h[\cdot] = \sum_h \pi^h[\cdot]$
  - adjusted Pareto weight: $\beta^h = \frac{\lambda^h}{\lambda} \frac{\partial V^h}{\partial I}$
- we arrive at the condition (where X_j is aggregate demand of good j)

$$\mathbb{E}_h[\sum_k t_k \frac{\partial h_j^h}{\partial q_k}] = X_j \mathbb{E}_h[\frac{x_j^h}{X_j}(-1 + \beta^h + \sum_k t_k \frac{\partial x_k^h}{\partial I})]$$

- Note that if we have homothetic and separable preferences then x_j^h / X_j is independent of j and from here we will see a uniform tax result.

Uniform Taxation

- If we have a lump sum tax then set such as:

$$\mathbb{E}_h[-1 + \beta^h + \sum_k t_k \frac{\partial x_k^h}{\partial I}] = 0$$

so we can write

$$\mathbb{E}_h[\sum_k t_k \frac{\partial h_j^h}{\partial q_k}] = X_j Cov_h[\frac{x_j^h}{X_j}, \hat{\beta}^h]$$

where $\hat{\beta}^h = -1 + \beta^h + \sum_k t_k \frac{\partial x_k^h}{\partial I}$.

- We get two intuitive cases:
  - $\hat{\beta}^h$ is constant;
  - x_j^h / X_j is independent of j, then back to single-agent case.
- Pareto inefficiency? If $\#agents\ H < \#goods\ N$ maybe cannot find welfare weights β^h that solve these equations
- **Uniform taxation:** Suppose utility is $U^h(G(x_1, \ldots, x_{N1}), H(x_{N1+1}, \ldots, x_N))$ and G, H are homogeneous of degree 1, then again it is optimal to tax uniformly within each group of goods. [treat each group of goods as inputs into production of G and H.]

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