Lecture 5: Dynamic Taxation I: Capital Taxation

Abdoulaye Ndiaye

NYU

Taxation with Idosyncratic shocks (Aiyagari, JPE 1995)

- infinitely lived household
- Each household receives an idiosyncratic shock θ_t that follows some non-degenerate Markov process.
- Let $\theta^t = (\theta_1, \ldots, \theta_t)$ is a history of shocks that each household received up to period t
- Households solve

$$v(b_0, \theta_0; \{\tau_t^k, \tau_t^l\}) = \max_{c,k,b,l} \mathbb{E}_0 \sum \beta^t u(c_t, l_t)$$

  s.t.

$$c(\theta^t) + k_{t+1}(\theta^t) + b_{t+1}(\theta^t) \leq (1 - \tau_t^l) w_t \theta_t l_t(\theta^t) + (1 - \tau_t^k)(1 + r_t - \delta)(k_t(\theta^{t-1}) + b_t(\theta^{t-1}))$$

$$k_{t+1}(\theta^t) + b_{t+1}(\theta^t) \geq D_{t+1}(\{\tau_t^k, \tau_t^l\})$$

- Here $D_{t+1}(\{\tau_t^k, \tau_t^l\})$ is the smallest present value of income that a household can make from period $t+1$ onwards.

Firms and Government

- Firms

$$\max F(K_t, L_t) - r_t K_t - w_t L_t$$

- Government picks $\{\tau_t^k, \tau_t^l, B_t, g_t\}$ to maximize

$$\mathbb{E}_0 \sum \beta^t u(c_t, l_t) + U(g_t)$$

with budget constraint

$$g_t + B_t = \tau_t^k (1 + r_t - \delta)(K_t + B_t) + \tau_t^l w_t L_t$$

- Feasibility

$$\int b_t(\theta^t) dF(\theta^t) + B_t = 0$$

$$\int \theta_t l_t(\theta^t) dF(\theta^t) = L_t$$

$$\int k_t(\theta^t) dF(\theta^t) = K_t$$

$$\int c_t(\theta^t) dF(\theta^t) + g_t + K_{t+1} = F(K_t, L_t) + (1 - \delta) K_t$$

- Redefine variables $\bar{w}_t \equiv (1-\tau_t^l)w_t, \bar{r}_t \equiv (1-\tau_t^k)(1+r_t-\delta), a_t \equiv k_t + b_t$

- Then the Ramsey planner can maximize over $\{w_t, r_t, \bar{w}_t, \bar{r}_t, K_t, B_t, L_t, a_t(\theta^t), c_t(\theta^t), l_t(\theta^t)\}$ instead of $\{w_t, r_t, \tau_t^l, \tau_t^k, K_t, B_t, L_t, b_t(\theta^t), k_t(\theta^t), c_t(\theta^t), l_t(\theta^t)\}$

- Households problem

$$v(a_0, \theta_0; \{\tau_t^k, \tau_t^l\}) = \max_{c,a,l} \mathbb{E}_0 \sum \beta^t u(c_t, l_t)$$

s.t.

$$c(\theta^t) + a_{t+1}(\theta^t) \leq \bar{w}_t \theta_t l_t(\theta^t) + \bar{r}_t a_t(\theta^{t-1})$$
$$a_{t+1}(\theta^t) \geq D_{t+1}(\{\bar{r}_t, \bar{w}_t\})$$

- We define optimal choices recursively as

$$c_t = c_t(a_t, \theta_t; \{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})$$
$$l_t = l_t(a_t, \theta_t; \{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})$$
$$a_{t+1} = a_{t+1}(a_t, \theta_t; \{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})$$

Household Problem

- This allows us to define functions

$$L_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty}), C_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty}), B_t = A_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty}) + K_t$$

- Important: if we have $\{a_t(\theta^t)\}_{\theta^t}, K_t$, then any $\{k_t(\theta^t), b_t(\theta^t)\}_{\theta^t}$ distribution that satisfies $k_t(\theta^t) + b_t(\theta^t) = a_t(\theta^t)$ and $\int k_t(\theta^t) = K_t$ is an equilibrium which gives the same welfare

- Therefore we can independently pick $\{a_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})\}$ and K_t, and can back out $\{b_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})\}, \{k_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})\}$ and government debt $\{B_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})\}$ as residuals.

Step back

- So far, not much progress
  - we have no idea how functions $\{L_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})\}, \{C_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})\}, \{A_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})\}$ depend on $\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty}$
- We could proceed as before and take all the FOCs but it is very hard to get any insight
  - we have infinitely many histories θ^t and corresponding constraints
  - distribution of asset holdings $\{a_t\}$ and as a result policy rules $c_t(a_t, \theta_t; \{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})$, etc are very hard to characterize
- This is a very general problem with incomplete market models
  - distribution of asset holdings and labor supply depend on current and future prices in a complicated way
  - prices are endogenous, function of the distribution of asset holdings and labor supply
  - even without taxation, equilibrium is a complicated fixed point problem which is hard to solve even numerically (see Krusell and Smith (JPE, 1998))
  - here, in addition we impose an outer layer of choosing optimal taxes

Partial characterization

- Lett's consider a partial characterization. Suppose the planner took $l_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty}), c_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})$ as given
- Ramsey problem is

$$\max_{\bar{r},\bar{w},K,g} \mathbb{E}_0 \sum \beta^t u(c_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty}), l_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})) + U(g_t)$$

s.t

$$C_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty}) + g_t + K_{t+1} = F(K_t, L_t(\{\bar{r}_t, \bar{w}_t\}_{t=0}^{\infty})) + (1-\delta)K_t$$

- Let's characterize capital taxation in the steady state

First order conditions

- $[g_t] : \beta^t U'(g_t) = \lambda_t$
- $[K_{t+1}] : \lambda_t = (1 + F_k(t+1) - \delta)$
- In steady state: prices and aggregate quantities are constant, individual quantities are not.
- In steady state aggregate capital stock is "undistorted"

$$(1 + F_k(ss) - \delta) = \frac{1}{\beta}$$

Implication for taxes

- Remember consumer's problem

$$v(a_0, \theta_0; \{\tau_t^k, \tau_t^l\}) = \max_{c,a,l} \mathbb{E}_0 \sum \beta^t u(c_t, l_t)$$

s.t.

$$c(\theta^t) + a_{t+1}(\theta^t) \leq \bar{w}_t \theta_t l_t(\theta^t) + \bar{r}_t a_t(\theta^{t-1})$$

$$a_{t+1}(\theta^t) \geq D_{t+1}(\{\bar{r}_t, \bar{w}_t\})$$

- FOCs w.r.t a_{t+1} imply Euler equation

$$u_c(t) \geq \bar{r} \beta \mathbb{E}_t u_c(t+1)$$

Martingale Convergence Theorem

- Martingale convergence theorem: one of the most important mathematical results for characterization of long-run distributions
- Stochastic process M_t is a
  - martingale if $M_t = M_{t+1}$ for all t
  - submartingale if $M_t \leq \mathbb{E}_t M_{t+1}$ for all t
  - supermartingale if $M_t \geq \mathbb{E}_t M_{t+1}$ for all t

Theorem

Suppose M_t is a martingale (or submartingale, or supermartingale) such that $sup_t \mathbb{E}_0 |M_t| < \infty$ (the expectations are bounded). Then M_t converges almost surely to a finite limit.

Application of MCT

- Suppose $\bar{r} \geq 1/\beta$. Then
$$u_c(t) \geq \mathbb{E}_t u_c(t+1)$$
- Therefore, $u_c(T)$ is a supermartingale.
- $u_c(t)$ is a bounded supermartingale since $u_c(t) \geq 0$ implies
$$u_c(0) \geq \mathbb{E}_t u_c(t+1) \geq 0$$
- Therefore, if $\bar{r} \geq 1/\beta$ then $u_c(t)$ converges to a finite limit.

Implications I

- Suppose that $u_c(t) \to u_c^* > 0$. Then $c_t \to c^* < \infty$. This c^* should satisfy a budget constraint. For $\varepsilon > 0, \exists T, \forall t \geq T$

$$c^* + a_{t+1}(\theta^t) \leq \bar{w}_t \theta_t l_t(\theta^t) + \bar{r}_t a_t(\theta^{t-1})$$

- DIfferent realizations of θ_t will lead to different l_t and different $\bar{w}_t \theta_t l_t(\theta^t)$

- Consider a long sequence of high and low realizations of$\bar{w}_t \theta_t l_t(\theta^t)$ so a_t should diverge to $\pm\infty$ which would violate the natural debt limit or transversality constraint. (or TVC)

Implications II

- Alternatively, suppose that $u_c(t) \to 0$ so that c_t diverges to ∞
- Then it must be true that a_t diverges to infinity
- Since
$$K_t + B_t = \int a_t$$
it must be true that $K_t + B_t$ diverges to infinity
- but K_t is bounded because there is a maximum sustainable capital stock in the economy
- B_t is bounded by present value of tax revenues on capital and labor
- Which leads to a contradiction.

Implications III

- This arguments show that $u_c(t)$ cannot be a martingale, and therefore

$$\bar{r} < 1/\beta$$

- This implies at a steady state

$$\begin{aligned} 1 + F_k(K_s s, L_s s) - \delta &= 1/\beta \\ &> \bar{r} = (1 - \tau_{ss}^k)(1 + r_{ss} - \delta) \\ &= (1 - \tau_{ss}^k)(1 + f_K(K_{ss}, L_{ss}) - \delta) \end{aligned}$$

- Therefore the optimal tax on capital is positive

$$\tau_{ss}^k > 0$$

Discussion

- In the presence of uninsurable idiosyncratic shocks, consumer has a precautionary motive to save, in addition to the regular consumption smoothing forces
    - for any finite amount of assets there is always a positive probability of hitting borrowing limit eventually
    - if $\bar{r} \geq 1/\beta$ consumer faces no (first order) losses by slightly increasing his savings and moving away from borrowing limit
    - finite distribution of assets is possible only with $\bar{r} < 1/\beta$
- Ramsey planner wants to be able to transfer resource between the periods with the shadow cost being equal to rate of discounting, which lead to $1 + F_k - \delta = 1/\beta$

The original result

- W. Rogerson (1985), "Repeated Moral Hazard," Econometrica 53 (1985), 69–76.
- Moral hazard model.
- Two periods $t = 0, 1$
  - effort in first period e_0
  - stochastic output in second period $y_1 = \theta_1$ with density $f(\theta_1|e_0)$
  - consumption in both periods c_0 and $c_1(\theta_1)$
  - linear savings technology with rate of return R
- Separable utility

$$u(c_0) - h(e_0) + \beta \int u(c_1(\theta_1))f(\theta_1|e_0)$$

- Incentive compatibility of $\{c_0, e_0, c_1(\theta_1)\}$ requires

$$u(c_0) - h(e_0) + \beta \int u(c_1(\theta_1))f(\theta_1|e_0) \geq u(c_0) - h(e_0') + \beta \int u(c_1(\theta_1))f(\theta_1|e_0')$$

Planning Problem

- Planning Problem

$$\min C(u_0) + \frac{1}{R}\int [C(u_1(\theta_1)) - \theta_1] f(\theta_1|e_0)$$

s.t. incentive compatibility and promise-keeping

$$u_0 - h(e_0) + \beta \int u(c_1(\theta_1)) f(\theta_1|e_0) \geq u_0 - h(e_0') + \beta \int u(c_1(\theta_1)) f(\theta_1|e_0')$$

$$u_0 - h(e_0) + \beta \int u(c_1(\theta_1)) f(\theta_1|e_0) \geq \underline{\mathrm{U}}$$

- If agents could save at R then we would have the Euler equation

$$u'(c_0) = \beta R \int u'(c_1(\theta_1)) f(\theta_1|e_0)$$

- We will show that this equation does not hold at the solution of the planning problem: there are savings distortions.

Proof

- Fix e_0 and consider variations in consumption/utility:

$$\hat{u}_0 = u_0 - \beta\Delta$$

$$\hat{u}_1(\theta_1) = u_1(\theta_1) + \Delta$$

- Preserves utility and incentive compatibility since for all e_0' we have

$$\hat{u}_0 - h(e_0) + \beta \int u(c_1(\theta_1)) f(\theta_1|e_0) = u_0 - h(e_0) + \beta \int u(c_1(\theta_1)) f(\theta_1|e_0)$$

- The optimal allocation must be immune to such variations

$$0 = arg \min_{\Delta} C(u_0 - \beta\Delta) + \frac{1}{R} \int [C(u_1(\theta_1) + \Delta) - \theta_1] f(\theta_1|e_0)$$

- Note a similarity with a savings problem, where Δ looks like an asset, $-C(-x)$ looks like a utility function, β^{-1} is the interest rate and R is the discount factor.

Proof

- The associated (FOC evaluated at $\Delta = 0$ is

$$C'(u_0) = \frac{1}{\beta R} \int C'(u_1(\theta_1))f(\theta_1|e_0$$

i.e

$$\frac{1}{u'(c_0)} = \frac{1}{\beta R} \int \frac{1}{u'(c_1(\theta_1))} f(\theta_1|e_0)$$

This is the Inverse Euler equation.

- As long as the variance of $c1(\theta_1)$ is strictly positive, then concavity of marginal utility and Jensen's inequality implies that the Euler equation is violated

$$u'(c_0) < \beta R \int u'(c_1(\theta_1))f(\theta_1|e_0)$$

- Agents are "savings-constrained". We have

$$u'(c_0) = \beta R(1 - \tau^s) \int u'(c_1(\theta_1))f(\theta_1|e_0)$$

with a positive savings wedge (implicit tax on savings/capital) $\tau^s > 0$

Application to Dynamic Mirrlees Model

- M. Golosov, N. Kocherlakota, and A. Tsyvinski, "Optimal Indirect and Capital Taxation," Review of Economic Studies 70 (2003), 569-587.
- Two periods $t = 0, 1$
  - work in second period $y_1(\theta_1)$
  - consumption in both periods c_0 and $c_1(\theta_1)$
  - linear savings technology with rate of return R
  - Separable utility
$$u(c_0) + \beta \int [u(c_1(\theta_1)) - h(y_1(\theta_1), \theta_1)] f(\theta_1 | e_0)$$
  - Similar planning problem...same optimality condition, Inverse Euler equation
$$\frac{1}{u'(c_0)} = \frac{1}{\beta R} \int \frac{1}{u'(c_1(\theta_1))} f(\theta_1 | e_0)$$
Same implications for the savings wedge.

Discussion

- The uncertainty in types over time and the inability to control labor supply force the planner to impose a distortion on savings to improve the provision of incentives to work.
- Another way to see this is that the desired labor supply at the optimal allocation is incompatible with free savings.
- Increasing savings in period t increases disposable income in period $t+1$.
- Unless utility is quasilinear, this implies an income effect on labor supply, and the agent is then tempted to work less.
- More savings in period t and lower labor supply in period $t+1$ are complements.
- Ruling out such a deviation requires discouraging savings below the level that would occur at the free-market rate.
- Quantitatively savings wedge small, and most of the insurance done with labor wedge... that will be our next class.

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