Lecture 6: Dynamic Taxation II: Income Taxation

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A Dynamic Life-Cycle Model

- Agents live for T years, work and consume
- Agents who work l_t hours make income $y_t = \theta_t l_t$.
- θ_t wage or productivity. Markov process with transition function $f^t(\theta_t|\theta_{t-1})$
- Planner observes c_t, y_t but not θ_t or l_t
- per-period utility

$$u(c_t, y_t; \theta_t) = u(c_t) - h(\frac{y_t}{\theta_t})$$

- lifetime utility from allocations

$$U(\{c(\theta^t), y(\theta^t)\}) = \sum_{t=1}^{T} \int \beta^{t-1}[u(c(\theta^t)) - h(\frac{y(\theta^t)}{\theta_t})]P(\theta^t)d\theta^t$$

$P(\theta^t) = f^t(\theta_t|\theta_{t-1}) \ldots f^2(\theta_2|\theta_1)f^1(\theta_1)$ and $d\theta^t = d\theta_t \dot{d}\theta_1$

Incentive Compatibility in a dynamic setting

- For a reporting strategy $r = \{r_t(\theta^t)\}_{t=1}^{T}$, continuation value after history θ^t

$$w^r(\theta^t) = u(c(r^t(\theta^t))) - h(\frac{y(r^t(\theta^t))}{\theta_t}) + \beta \int w^r(\theta^{t+1}) f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

- Continuation value under truthful revelation

$$w(\theta^t) = u(c(\theta^t)) - h(\frac{y(\theta^t)}{\theta_t}) + \beta \int w(\theta^{t+1}) f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

- IC means truth-telling yields higher continuation utility than any other reporting strategy

$$(\text{IC}) : w(\theta_1) \geq w^r(\theta_1) \quad \forall \theta_1, \forall r$$

- Consider reporting strategy $\tilde{r}^t(\theta^t) = (\theta^{t-1}, \theta')$

$$w^{\tilde{r}}(\theta^t) = u(c(\theta^{t-1}, \theta')) - h(\frac{y(\theta^{t-1}, \theta')}{\theta_t}) + \beta \int w^{\tilde{r}}(\theta^{t-1}, \theta', \theta_{t+1}) f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

- IC implies

$$w(\theta^t) = \max_{\theta'} w^{\tilde{r}}(\theta^t)$$

First Order Approach

- Envelope condition of the agent, which is necessary for incentive compatibility

$$\frac{\partial w(\theta^t)}{\partial \theta_t} = -\frac{y(\theta^t)}{(\theta_t)^2} h'(\frac{y(\theta^t)}{\theta_t}) + \beta \int w(\theta^{t+1}) \frac{\partial f^{t+1}(\theta_{t+1}|\theta_t)}{\partial \theta_t} d\theta_{t+1}$$

- The planner's objective is to minimize the expected discounted cost of providing an allocation

$$\min_{c,y} \sum_{t=1}^{T} (1/R)^{t-1} \int (c(\theta^t) - y(\theta^t)) P(\theta^t) d\theta^t$$

- subject to expected lifetime utility of each (initial) type θ_1 being above a threshold

$$U(\{c, y\}; \theta_1) \geq \underline{\text{U}}(\theta_1)$$

- And incentive compatibility replaced by the necessary envelope condition: First-Order Approach

- Numerically verify global IC ex-post

Recursive Formulation of the Relaxed Program

- Denote expected continuation utility

$$v(\theta^t) \equiv \int w(\theta^{t+1}) f^{t+1}(\theta_{t+1}|\theta_t) d\theta_{t+1}$$

- Continuation utility $w(\theta^t)$ can be rewritten as

$$w(\theta^t) = u(c(\theta^t)) - h(\frac{y(\theta^t)}{\theta_t}) + \beta v(\theta^t)$$

- With persistence, planner needs also to control variation in continuation value with type

$$\Delta(\theta^t) \equiv \int w(\theta^{t+1}) \frac{\partial f^{t+1}(\theta_{t+1}|\theta_t)}{\partial \theta_t} d\theta_{t+1}$$

- The envelope condition can then be rewritten as

$$\frac{\partial w(\theta^t)}{\partial \theta_t} = \frac{y(\theta^t)}{(\theta_t)^2} h'(\frac{y(\theta^t)}{\theta_t}) + \beta \Delta(\theta^t)$$

- State variable $(v_{t-1}, \Delta_{t-1}, \theta_{t-1}, t)$

Recursive Planning Problem

$$K(v, \Delta, \theta_-, t) = \min_{c(\theta), y(\theta), w(\theta), v(\theta), \Delta(\theta)} \int (c(\theta) - y(\theta) + \frac{1}{R} K(v(\theta), \Delta(\theta), \theta, t+1)) f^t(\theta|\theta_-) d\theta$$

subject to

$$w(\theta) = u(c(\theta)) - h(\frac{y(\theta)}{\theta_t}) + \beta v(\theta)$$

$$\dot{w}(\theta) = \frac{y(\theta)}{(\theta)^2} h'(\frac{y(\theta)}{\theta}) + \beta \Delta(\theta)$$

$$v = \int w(\theta) f^t(\theta|\theta_-) d\theta$$

$$\Delta = \int w(\theta) \frac{\partial f^t(\theta|\theta_-)}{\partial \theta_-} d\theta$$

Solution Method: Hamiltonian

- Denote λ and γ the multipliers on the third and fourth constraints

$$K_v(v, \Delta, \theta_-, t) = \lambda, \quad K_\Delta(v, \Delta, \theta_-, t) = \gamma$$

- In line with these identities, we write

$$K_v(v(\theta), \Delta(\theta), \theta, t+1) = \lambda(\theta), \quad K_\Delta(v(\theta), \Delta(\theta), \theta, t+1) = \gamma(\theta)$$

- Denote $\mu(\theta)$ the co-state variable associated with $w(\theta)$. The Hamiltonian is

$$[C^t(y(\theta), w(\theta) - \beta v(\theta), \theta) - y(\theta)] f^t(\theta|\theta_-)$$
$$+ \frac{1}{R} \int K(v(\theta), \Delta(\theta), \theta', t+1) f^{t+1}(\theta'|\theta) d\theta' f^t(\theta|\theta_-)$$
$$+ \lambda[v - w(\theta) f^t(\theta|\theta_-)] + \gamma[\Delta - w(\theta) \frac{\partial f^t(\theta|\theta_-)}{\partial \theta_-}]$$
$$+ \mu(\theta)[u_\theta(C^t(y(\theta), w(\theta) - \beta v(\theta), \theta), y(\theta), \theta) + \beta \Delta(\theta)]$$

with the boundary conditions

$$\lim_{\theta \to \bar{\theta}} \mu(\theta) = 0 \text{ and } \lim_{\theta \to \underline{\theta}} \mu(\theta) = 0$$

Wedges

- Solution to the relaxed program characterized with wedges, implicit taxes and subsidies
- Intratemporal wedge

$$\tau_L(\theta^t) \equiv 1 + \frac{h'(l_t)}{\theta_t u'(c_t)}$$

- Intertemporal or savings wedge

$$\tau_K(\theta^t) \equiv 1 - \frac{1}{R\beta} \frac{u'(c_t)}{\mathbb{E}_t u'(c_{t+1})}$$

First order conditions

- The law of motion for the co-state $\mu(\theta)$ is (with $g^t(\theta|\theta_-) = \frac{\partial f^t(\theta|\theta_-)}{\partial \theta_-}$)

$$\frac{d\mu(\theta)}{d\theta} = -[\frac{1}{u'(c(\theta))} - \lambda - \gamma \frac{g^t(\theta|\theta_-)}{f^t(\theta|\theta_-)}]f^t(\theta|\theta_-)$$

- And the FOCs for $\Delta(\theta), v(\theta)$ and $y(\theta)$ are

$$\frac{\mu(\theta)}{\theta f^t(\theta|\theta_-)} = -\frac{1}{R\beta}\frac{\gamma(\theta)}{\theta}$$

$$\frac{1}{u'(c(\theta))} = \frac{1}{\beta R}\lambda(\theta)$$

$$1 - \frac{h_y(\frac{y(\theta)}{\theta})}{u'(c(\theta))} = \frac{\mu(\theta)}{f^t(\theta|\theta_-)}[-h_{y\theta}(\frac{y(\theta)}{\theta})]$$

Inverse Euler Equation

- Integrating LoM of co-state and replacing λ from the FOC for $v(\theta)$ yields another proof of the IEE.

$$0 = \int [\frac{1}{u'(c(\theta))} - \frac{\beta R}{u'(c_-)}] f^t(\theta|\theta_-) d\theta$$

- The intertemporal wedge satisfies

$$\tau_k(\theta^{t-1}) = 1 - \frac{[\int [u'(c(\theta^t))]^{-1} f^t(\theta_t|\theta_{t-1}) d\theta_t]^{-1}}{\int u'(c(\theta^t)) f^t(\theta_t|\theta_{t-1}) d\theta_t}$$

- By Jensen's inequality $\tau_K > 0$
- Positive savings distortions are present at the constrained optimum.

Labor Wedge

- Assume h isoelastic $h(l) = \frac{l^{1+1/\varepsilon}}{1+1/\varepsilon}$
- Assume log autoregressive productivity process with persistence ρ

$$\log(\theta_t) = \rho \log(\theta_{t-1}) + \epsilon_t$$

- Then the labor wedge satisfies

$$\mathbb{E}_{t-1}[\frac{\tau_{L,t}}{1-\tau_{L,t}} \frac{1}{R\beta} \frac{u'(c_{t-1})}{u'(c_t)}] = \rho \frac{\tau_{L,t-1}}{1-\tau_{L,t-1}} + (1+\frac{1}{\varepsilon})\mathrm{Cov}(\log(\theta_t), \frac{1}{R\beta} \frac{u'(c_{t-1})}{u'(c_t)})$$

Intuition

- Labor wedge formula

$$\mathbb{E}_{t-1}[\frac{\tau_{L,t}}{1-\tau_{L,t}}\frac{1}{R\beta}\frac{u'(c_{t-1})}{u'(c_t)}] = \rho\frac{\tau_{L,t-1}}{1-\tau_{L,t-1}} + (1+\frac{1}{\varepsilon})\mathrm{Cov}(\log(\theta_t), \frac{1}{R\beta}\frac{u'(c_{t-1})}{u'(c_t)})$$

- LHS: risk-adjusted conditional expectation of $\frac{\tau_{L,t}}{1-\tau_{L,t}}$
- RHS(1): $\frac{\tau_{L,t}}{1-\tau_{L,t}}$ inherits persistence of $\{\theta\}$
- RHS(2): positive drift of $\frac{\tau_{L,t}}{1-\tau_{L,t}}$
  - benefit of added insurance $\mathrm{Cov}(\log(\theta_t), \frac{1}{R\beta}\frac{u'(c_{t-1})}{u'(c_t)})$
  - incentive cost increases with elasticity ε
- With random walk productivity labor wedge increase with age

Numerical Simulation

- Agents live for $T = 60$ years work for 40 years and retire for 20 years
- Utility function

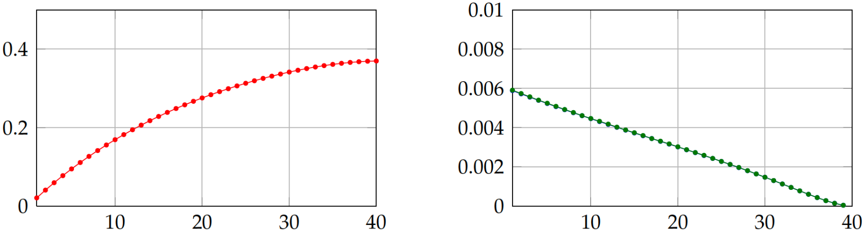
$$\log(c_t) - \frac{l^{1+1/\varepsilon}}{1+1/\varepsilon}$$

with $\varepsilon = 0.5, 1/R = \beta = 0.95$

- Productivity process: random walk

$$\theta_t = \varepsilon_t \theta_{t-1}$$

Age-dependence of Wedges



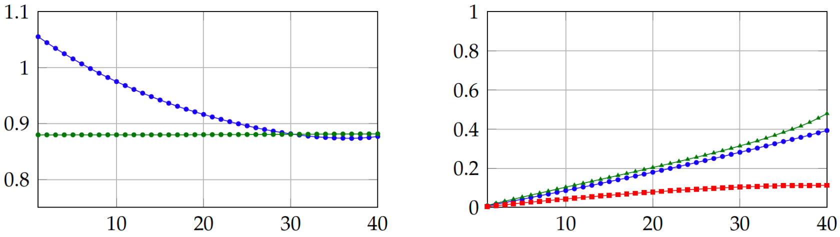
(a) Average labor wedge over time.

(b) Intertemporal wedge and variance of consumption growth over time—both series are indistinguishable to the naked eye.

Figure 1: Average wedges over time.

- as retirement nears uncertainty goes to 0
- labor tax increasing over time $\implies$ increased insurance

Optimal Allocations



(a) Average for output (declining line) and consumption (constant line) over time.

(b) Variance of output (top line), productivity (middle line) and consumption (bottom line) over time.

Figure 2: Statistics for optimal allocation over time.

- Consumption smoothing. Output declining over time
- Variance of consumption less than variance of productivity: insurance

Tax Smoothing

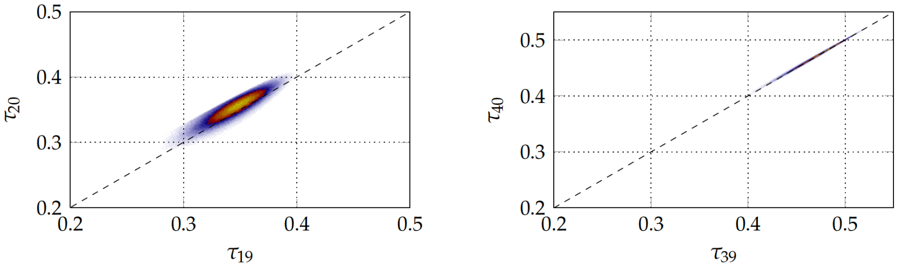


Figure 3: Scatter plot of $\tau_{L,t}$ vs. $\tau_{L,t-1}$ at $t = 20$ and $t = 40$.

- Tax smoothing: slope close to one
- dispersion: innovations in c_t
- Drift: above 45 degree line
- Near retirement: lower dispersion, smaller drift

Insurance

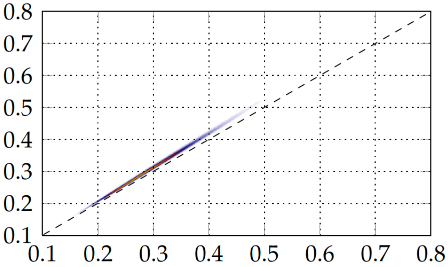
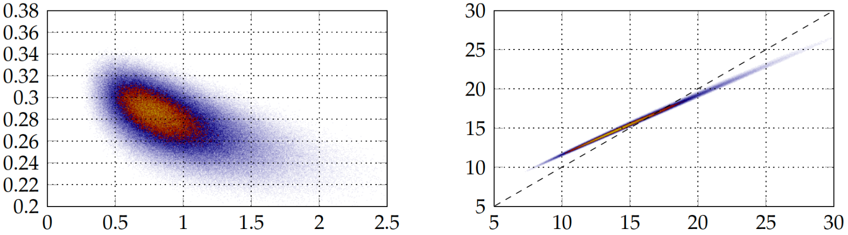


Figure 4: Scatter plot of $\frac{\tau_{L,t}}{1-\tau_{L,t}}u'(c_t)$ against $\frac{\tau_{L,t-1}}{1-\tau_{L,t-1}}u'(c_{t-1})$ for $t = 20$.

History-Dependence and Insurance



(a) Scatter plot of $\tau_{L,t}$ vs. θ_t at $t = 20$.

(b) Scatter plot of $\sum_{t=1}^{T} q^t c_t$ vs. $\sum_{t=1}^{T} q^t y_t$.

Figure 5: History dependence and Insurance

- Regressive tax on average: short-term regressivity
- History dependence: dispersion
- Insurance: slope of 0.67

Welfare Analysis

	$\hat{\sigma}^2 = 0.0061$	$\hat{\sigma}^2 = 0.0095$	$\hat{\sigma}^2 = 0.0161$
second-best	0.86%	1.56%	3.43%

Table 1: Welfare gains over free-savings, no-tax equilibrium.

	$\hat{\sigma}^2 = 0.0061$	$\hat{\sigma}^2 = 0.095$	$\hat{\sigma}^2 = 0.0161$
age-dependent τ_L and τ_K	0.71%	1.47%	3.30%
age-dependent τ_L, and $\tau_K = 0$	0.66%	1.38%	3.16%
age-dependent τ_L, age-independent τ_K	0.70%	1.46%	3.29%
age-independent τ_L and τ_K	0.54%	1.14%	2.71%

Table 2: Welfare from simple tax policies: history-independent (linear) but possibly age-dependent taxes.

- linear taxes = cross-sectional average wedges from simulation
- bulk of welfare gains achieved by linear age-dependent policies

Bibliography

Farhi, E., and Werning, I. (2013). "Insurance and Taxation over the Life Cycle." *Review of Economic Studies*, 80(2), 596–635.

Golosov, M., Troshkin, M., and Tsyvinski, A. (2016). "Redistribution and Social Insurance." *American Economic Review*, 106(2), 359–386.

Stantcheva, S. (2017). "Optimal Taxation and Human Capital Policies over the Life Cycle." *Journal of Political Economy*, 125(6), 1931–1990.

Ndiaye, A. (2020). "Flexible Retirement and Optimal Taxation." FRB of Chicago Working Paper No. WP-2018-18 .