Lecture 9: Taxation and Aggregate Shocks

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Barro (JPE, 1979)

- Barro, Robert J. 1979. On the determination of the public debt. Journal of Political Economy 87(5): 940-971
- Reduced form model, PE model, no capital accumulation (so no capital taxation).
- Governmentt uses distorting taxes to finance stochastic g_t
- $D(\tau)$ is the deadweight loss
- Government problem

$$\min_{\tau} \sum_{t=0}^{\infty} (1+r)^{-t} D(\tau_t)$$

s.t

$$g_t + b_{t+1} = \tau_t + (1+r) b_t$$

Tax smoothing

- If no uncertainty: for all t

$$\tau_t = \tau_{t+1}$$

- if there is uncertainty: for all t

$$D'(\tau_t) = E_t D'(\tau_{t+1})$$

- If deadweight loss is quadratic (which is true to the first order), taxed are random walk

$$\tau_t = E_t \tau_{t+1}$$

- From government b.c. debt is random walk

$$b_t = E_t b_{t+1}$$

- Intuition parallels that behind Friedman's permanent income hypothesis results.

Lucas and Stokey (JME, 1983)

- Lucas, Robert Jr. & Stokey, Nancy L., 1983. "Optimal fiscal and monetary policy in an economy without capital," Journal of Monetary Economics, Elsevier, vol. 12(1), pages 55-93.
- Model: g_t is an exogenous Markov process.
- No capital, linear technology
- Representative consumer.
- General equilibrium
- Complete markets

Consumer's budget constraint

$$\max E \sum \beta^t u(c(s^t), l(s^t))$$

s.t

$$c(s^t) + \sum_{s^{t+1} \geq s^t} q(s^{t+1}|s^t) a(s^{t+1}) \leq (1 - \tau(s^t)) w(s^t) l(s^t) + a(s^t)$$

$$a(s^0) = a_0$$

where s^t is a history of realizations of g^t , a is Arrow security,a_0 is initial wealth (vis-a-vis government).

Similarly the government also uses a state-contingent debt.

Implementabilibty

Can re-write the expression above as

$$\sum_{t,s^t} q(s^t)c(s^t) \leq \sum_{t,s^t} q(s^t)(1-\tau(s^t))w(s^t)l(s^t)$$

Not surprisingly, the implementability constraint becomes

$$\sum_{t,s^t} [u_c(s^t)c(s^t) + u_l(s^t)l(s^t)] = u_c(s_0)a_0$$

Plus feasibility

$$c(s^t) + g(s^t) \leq Al(s^t)$$

Take the FOCs wrt to c and l at $s^t > s^0$

$$[u_{cc}(s^t)c(s^t) + u_c(s^t) + u_{lc}(s^t)l(s^t)]\eta = \lambda(s^t)$$

$$[u_{cl}(s^t)c(s^t) + u_l(s^t) + u_{ll}(s^t)l(s^t)]\eta = -\lambda(s^t)A$$

- We have three unknown variables for state s^t, (c, l, λ) and three equations (FOCS + feasibility)
- The fourth unknown, η, is the same for all dates and states and is determined by the period 0 budget constraint.
- This implies that if in two different periods or states g is the same, then (c, l, λ) are the same as well.
- Since $w(s^t) = A$ and taxes are determined from MRS of consumers, this implies that taxes are the same for the same level of g. Thus, taxes must be smooths across all states.

- Note that prescription for taxes is also very different from Barro.
- If g is iid, the above result implies that taxes are iid also. There is no history dependence!
- The government should use state-contingent debt to smooth distortions across time.

Special case

Isoelastic preferences: $u(c,l) = \frac{c^{1-\sigma}}{-\sigma} - \alpha \frac{l^{\gamma}}{\gamma}$
In this case

$$[u_{cc}(s^t)c(s^t) + u_c(s^t) + u_{lc}(s^t)l(s^t)] = (1-\sigma)c^{-\sigma} = (1-\sigma)u'(c)$$

$$[u_{cl}(s^t)c(s^t) + u_l(s^t) + u_{ll}(s^t)l(s^t)] = \alpha\gamma l^{\gamma-1} = \gamma v'(l)$$

and the FOCs imply

$$\frac{u'(c)}{v'(l)} = -A \times constant$$

Thus,the tax is the same for all states with this utility (to reconcile with the above result, taxes are still "iid" but their variance is zero).

Time inconsistency

- Note that if $a_0 \neq 0$, then FOCs for period 0 are different because of the $u_c(s^0)$ term
  - The government has incentives to play with taxes in period 0 to reduce the market value of debt (increase market value of assets) it was born with
- No similar effect in other periods since forward looking agents take it into account
  - akin to capital tax that has no distortions in period 0
- Time consistency problem: if governmentt can re-optimize at future dates, it would have incentives to do so
  - same as in capital taxation

Aiyagari, Marcet, Sargent, Seppala (JPE, 2002)

- S. Rao Aiyagari & Albert Marcet & Thomas J. Sargent & Juha Seppala, 2002. "Optimal Taxation without State-Contingent Debt," Journal of Political Economy, University of Chicago Press, vol. 110(6), pages 1220-1254, December.
- Why did we get different insights in Barro and Lucas-Stokey?
- Model: g_t is an exogenous Markov process.
- No capital, linear technology
- Representative consumer.
- General equilibrium
- Incomplete markets: agents can only trade a risk-free bond
- AMSS also add that government can pay transfers $T(s^t) \geq 0$ (which I omit here)

- Consumers maximize

$$\max E_0 \sum \beta^t u(c(s^t), l(s^t))$$

subject to

$$c(s^t) + b(s^t) = (1 - \tau(s^t))w(s^t)l(s^t) + (1 + R(s^{t-1}))b(s^{t-1})$$

- Government budget constraint

$$g(s^t) + B(s^t) = \tau(s^t)w(s^t)l(s^t) + (1 + R(s^{t-1}))B(s^{t-1})$$

- Feasibility

$$c(s^t) + g(s^t) \leq Al(s^t)$$

$$b(s^t) + B(s^t) = 0$$

Necessary and sufficient conditions for consumers

1. Budget constraint

2. FOCs

3. TVC (see Magill and Quinzii (Econometrica, 1994) for proofs)

$$\lim_{T \to \infty} E[\beta^T u(c_T, l_T) b_T | s^t] = 0 \text{ for all } s^t$$

In AMSS we have a budget constraint in each period

$$c(s^t) + b(s^t) = (1 - \tau(s^t))w(s^t)l(s^t) + (1 + R(s^{t-1})b(s^{t-1})$$

Substitute the FOCs to get

$$u_c(s^t)c(s^t) + u_c(s^t)b(s^t) = -u_l(s^t)l(s^t) + u_c(s^t)(1 + R(s^{t-1}))b(s^{t-1})$$

The interest rate is

$$\frac{1}{1 + R(s^{t-1})} = \frac{\beta \sum_{s^t} Pr(s^t|s^{t-1})u_c(s^t)}{u_c(s^{t-1})}$$

Substitute that into the equation above to get

$$u_c(s^t)c(s^t) + u_c(s^t)b(s^t) = -u_l(s^t)l(s^t) + \frac{u_c(s^t)}{\beta \sum_{s^t} Pr(s^t|s^{t-1})u_c(s^t)} u_c(s^{t-1})b(s^{t-1})$$

Let $a(s^t) \equiv u_c(s^{t-1})b(s^t)$, so that the above equation becomes

$$u_c(s^t)c(s^t) + a(s^t) = -u_l(s^t)l(s^t) + \frac{u_c(s^t)}{\beta \sum_{s^t} Pr(s^t|s^{t-1})u_c(s^t)} a(s^{t-1})$$

Ramsey Problem

$$\max_{c,l,a} E_0 \sum \beta^t u(c(s^t), l(s^t))$$

subject to

$$u_c(s^t)c(s^t) + a(s^t) = -u_l(s^t)l(s^t) + \frac{u_c(s^t)}{\beta \sum_{s^t} Pr(s^t|s^{t-1})u_c(s^t)} a(s^{t-1})$$

$$c(s^t) + g(s^t) \leq Al(s^t)$$

and TVC

- Exercise:
  - take FOCs and get expressions for taxes
  - stare at them
  - convince yourself that it is hopeless... [usually the case with incomplete markets]
- Intuition:
  - generally, distortion from taxes depend not only on taxes but also on the asset holdings (income effects)
  - optimal asset position depends on the risk
- Let $\beta^t Pr(s^t)\psi(s^t)$ be Lagrange multiplier on the implementability constraint
  - use $\psi(s^t)$ as a measure of distortion in s^t

- FOCs for $a(s^t)$:

$$\psi_t = E_t[\frac{u_{c,t+1}}{E_t u_{c,t+1}}\psi_{t+1}]$$

- This implies that

$$\psi_t = E_t \psi_{t+1} + \frac{Cov_t[u_{c,t+1}, \psi_{t+1}]}{E_t u_{c,t+1}}$$

- Consider quasi-linear preferences

$$c_t - \frac{l^{1+1/\gamma}}{1+1/\gamma}$$

- They imply $u_c = 1, R_t = 1/\beta, Cov_t[u_{c,t+1}, \psi_{t+1}] = 0$

- Also $l_t^{1/\gamma} = 1 - \tau_t$

FOCs

FOCs are easy now:

$$[c_t]: \quad 1 + \psi_t = \lambda_t$$

$$[l_t]: \quad l_t^{1/\gamma}[1 + \psi_t \frac{1}{\gamma}] = \lambda_t$$

Therefore

$$\psi_t = E_t \psi_{t+1}$$

$$1 - \tau_t = -l_t^{1/\gamma} = \frac{1 + \psi_t}{1 - \psi_t \frac{1}{\gamma}}$$

Tax rates are not exactly random walk but quite similar to it

Bibliography

Aiyagari, S. R., Marcet, A., Sargent, T. J., and Seppälä, J. (2002). "Optimal Taxation without State-Contingent Debt." *Journal of Political Economy*, 110(6), 1220–1254.

Barro, R. J. (1979). "On the Determination of the Public Debt." *Journal of Political Economy*, 87(5), 940–971.

Gaur, M., Grigsby, J., Hazell, J., and Ndiaye, A. (2023). "Bonus Question: How Does Flexible Incentive Pay Affect Unemployment Dynamics?" IZA Discussion Paper No. 16481.

Lucas, R. E., Jr., and Stokey, N. L. (1983). "Optimal Fiscal and Monetary Policy in an Economy without Capital." *Journal of Monetary Economics*, 12(1), 55–93.